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REVIEWS

Rarefied Gas Dynamics. Edited by J. A. LAURMANN. Academic Press, 1963. Volume 1, 541 pp., £5. 14*s*. 6*d*. Volume 2, 529 pp., £5. 14*s*. 6*d*.

These two large volumes contain fifty-five papers contributed to the Third International Symposium on Rarefied Gas Dynamics, held in Paris in June 1962 under the chairmanship of E. A. Brun and I. Estermann.

Comparison with the Proceedings of the previous Symposia (1st, Nice, 1958; 2nd, Berkeley, 1960) shows the rapid progress in the field of rarefied gases, particularly in experimental work. The substantial growth of experimental data is connected with the fact that an increasing number of laboratories are now occupied with rarefied gas phenomena, which are evidently not of purely theoretical interest.

The papers are divided into six sections, but the boundary between the different sections is not sharp. The authors of a great majority of the theoretical papers of necessity use model descriptions or assume some simplifying conditions. It is evident that only a comparison with experimental data will show whether their assumptions are correct and the range of the validity of the proposed theories. On the other hand, the experimental work in rarefied gas dynamics is clearly led by theoretical predictions and is aimed at checking the existing theories and hypotheses. I think that the situation in rarefied gas theory is now closer to that in nuclear physics than to that in classical aerodynamics.

The first section deals with the 'Fundamentals of kinetic theory and solutions of the Boltzmann equations'. However, there are only two papers in which the Boltzmann equation is considered from the mathematical point of view. The main result of Grad's analysis is the proof of the existence of the solution of the Boltzmann equation. The author uses functional analysis and particularly the embedding theorems. Hence the solution is understood in the generalized sense. (Similar proofs have recently been found, for example, for the Navier–Stokes equation.) Some essential but physically meaningful requirements are imposed on the type of intermolecular potential. For some class of functions the proof of uniqueness of the solutions is also obtained.

The second paper on the full Boltzmann equation was presented by Janos who analysed the convergence of iterative solutions, obtained by successive linearization of the collision integral. As the starting approximation a collisionless solution was taken. The author found some satisfactory conditions for the convergence.

In Lunc's paper a measure of the degree of departure of an actual state from the equilibrium state is introduced. It is defined as the square of a normalized deviation function with the equilibrium distribution acting as a weight function. The classification corresponds in some cases with Tsien's classification of flow regimes.

Havilland presents an investigation of shock waves using the Monte Carlo method. The Monte Carlo approach may be considered as a 'mathematical experimental method' in which the molecular phenomena are simulated by a high speed computer. Assuming some initial distribution we allow the molecules to interact with one another. The accuracy of the result depends on the number of steps taken. The author obtained shock profiles for $M = 1{\cdot}5$, $2{\cdot}0$ and $3{\cdot}0$, assuming that the molecules behave as hard spheres and the results are compared with some known theoretical solutions.

A similar approach to rarefied gas dynamics is given by Schaetzle. However, in his method, instead of the individual molecules, groups of molecules are traced from location to location. As a first step the author assumes that the traced groups of molecules are colliding with molecules of Maxwellian distribution. The iteration process continues until the stable distribution is reached. The proposed iteration process is faster than that of the Monte Carlo method. The technique was applied to the free molecular flow region for a flat plate at zero angle of attack, to the slip flow region in Couette flow, and to the boundary layer at the leading edge of a flat plate. These results show good agreement with the experimental data and other theoretical results.

Suchy gives in his paper a general method for the treatment of phenomena far from the equilibrium state. He assumes the distribution function to be in the form of an integral superposition of Maxwellian distributions A distribution, with the weighting function containing a finite number of velocity independent parameters, is used for calculation of collision integrals in the moment equations. The author thus obtains the moment equations up to the third order.

Chahine considers the structure of a plane shock wave in a monatomic gas for $M = 1$ to 10. Krook's model is used and the appropriate equation is solved by an iterative method. Starting from the solution of the Navier–Stokes equation the author determined the velocity and temperature profiles.

Heat transfer between parallel plates at large temperature differences is the subject of Willis's paper. The main aim of the work is to check the accuracy of a different approximation used to solve the Boltzmann equation. As a basis for comparison the author uses Krook's model, which enables him to obtain accurate numerical results. Comparison is made between Lees & Liu's method (*Rarefied Gas Dynamics,* 1961) and his own method.

Sirovich and Thurber investigate sound propagation according to the kinetic theory of gases. To do this they use the model kinetic equation containing three relaxation times. This model, suggested by Gross & Jackson (*Phys. Fluids* **2**, 1959), can be considered as a generalization of the BGK-model which contains only one relaxation time. According to the authors, 'kinetic models are unable to describe phenomena of higher than a certain wave number, the latter depending on the model chosen'.

The paper presented by Scala and Talbot is devoted to the analysis of shock wave structure in a gas with two degrees of freedom. Rotation and vibration of a diatomic gas are considered for $M = 4{\cdot}6$ and 10. The assumption is made that relaxation times for the internal degrees of freedom are large compared with relaxation times for translation. The Navier–Stokes equations are used with the relaxation equations and the system was solved numerically with the aid of a high-speed computer. The results seem reasonable for the full range of relaxation times and Mach numbers dealt with.

Section 2 is devoted to molecular beams and surface interactions. The section is opened by a paper from McKeown on surface erosion in space. The measurements of the erosion of gold and silver were made on Discoveries 26 and 32. The sputtering was determined to be less than 5×10^{-6} atom/molecule. According to the authors, erosion presents no real problem to earth satellites travelling in interplanetary space.

In Nocilla's paper the surface re-emission law is studied. This paper is an extension of the one presented at the Second International Symposium. The basic idea is that the molecules re-emitted from the element behave as a gas in Maxwellian equilibrium with translational velocity. Hence it is possible to calculate the intensity of the flux of the re-emitted gas as a function of the solid angle. In the formula obtained two parameters occur which are chosen to give the best possible agreement between the theoretical and experimental data. There is good agreement except for glancing incidences. According to the author the theory may be extended to geometrically more complicated bodies.

An interesting paper on reflexion of helium and deuterium from a platinum surface is presented by Datz, Moore and Taylor. The most important results are as follows. The angular distribution conforms to two general patterns depending on the temperature of the surface: for high temperatures the distribution is close to the specular; and for room temperatures a diffuse distribution is observed. This may be attributed to the adsorption properties of the surface. Adding O_2 to the background gas altered the situation and the reflexion remained diffuse even at higher temperatures. On the other hand, the addition of H_2, which does not interact with Pt, did not change the observed situation.

Devienne and his co-workers describe a modified version of the molecular gun. It is designed for the investigation of the interaction between ion-beams and neutral beams (argon is used in both). The gun allows energies ranging from 500 to 3000 eV to be obtained, and a detailed analysis of the gun performance is given. In the same laboratory a revolving disk apparatus was built, which enables a wider class of phenomena to be studied, such as boundary layers in rarefied gases, holding-time of the molecules on the surface, dispersion of reflected molecules, etc. In spite of the fact that the principle of the apparatus is simple, its technical realization is difficult. The pressure of the reflected molecules is investigated using a special detecting system. When the disk is at rest the angular distribution curve is a circle (all experiments are performed at room temperature). For the moving disk the curves show a maximum for a given angle, the value of which depends on the speed of the disk and the nature of the gas. A simple theoretical description, based on the assumption that the horizontal component of the velocity is added to the average thermal velocity of the molecules, agrees qualitatively with the experimental results.

Jawtusch studies the reflexion of molecular beams of air, N_2 and Ar from Nikuradse's synthetic coatings. A smooth silver layer is used as a reference surface.

Using the modulated-atomic-beam technique Smith and Fite investigate the reflexion of molecular gases from polycrystalline nickel, the dissociation of molecular hydrogen at a tungsten surface and the reaction of molecular chlorine and nickel to form nickel chlorides. The results obtained for the reflexion of molecular gases from polycrystalline nickel are: (*a*) for target temperatures less than 20 °C and greater than 870 °C the scattering is diffuse; (*b*) the occurrence of carbon impurities in the nickel sample influences the scattering properties of the metal.

A new apparatus for measuring the transfer of normal momentum between a gas and metallic surface was built by Stickney and Hurlbut. The principle of the apparatus is to use a very sensitive torsion balance. It enables the precise absolute measurement of gas-surface interactions to be made. The results for several systems are reported (He, Ne, H, N, CO_2 on tungsten, aluminium, blackened tungsten and platinum). It was found that it is not possible to describe the surface interaction of the molecules as a combination of two components, one reflected specularly and the other diffusely. In some way the authors are giving a finite solution of the problem posed; their arguments look very convincing. The authors propose an alternative model of re-emission.

Using electron paramagnetic resonance spectroscopy for measuring the atomic density and calorimetry for measuring the energy flux, Wise and Wood were able to determine the energy accommodation coefficient. About 20 % of the energy released in the recombination of hydrygen atoms on nickel is transferred to the surface. Values of the energy accommodation during the recombination of hydrogen atoms on various metals is also reported.

The last three papers of this section are devoted to the investigation of aerodynamic molecular beams with aerodynamic nozzles. These devices were introduced by Kontorowitz & Gray in 1951. Two such apparatuses were constructed, one in Princeton University by Fenn & Deckers and the second in the University of Virginia by Scott & Drewry. They achieve a substantial fraction of the intensities predicted by theory. The velocity spread in the aerodynamic beams is smaller than in the standard thermal effusion beams. It is generally accepted that this kind of device enables fundamental measurements in rarefied gas dynamics to be made.

In the paper of Bier and Hagena the influence of shock waves occurring in high-intensity molecular beams is examined. To this group we may also add the paper of Muntz and Marsden, in which a new method for the measurement of number density is considered. It is based essentially on the observation of luminescence of the flowing gas excited by the electrons in the beam.

Section 4 deals with the theory of transition flow.

Cercignani investigated plane Poiseuille flow and the Knudsen minimum effect. He uses the linearization of the Boltzmann equation and the linearized form of the collision integral is taken in the form of the BGK model. Complete diffusion at the walls is assumed. The resulting linear integral equation is discussed and it is shown that the usual iteration procedure gives a result only for very low pressures. Starting from the differential equation and using his previous results the authors shows that the Knudsen minimum follows from his approach. According to the calculations performed on the basis of finite differences, this minimum occurs at $d/\lambda = 0{\cdot}77$, where d is the distance between two plates and λ is the mean free path, which is in good agreement with the experimental data.

Takao considers the problem of heat transfer from a sphere in a rarefied monatomic gas for the entire range of Knudsen numbers. The paper is an extension of the method which has been introduced previously by the author and by Fujimoto. To generalize the results to include air a correction factor is introduced. The theoretical results are in very good agreement with the experimental data obtained by the author.

As Shen says, 'the assumed distribution function in addition to agreeing with the free molecule and continuum flows in the respective limits, must also exhibit at least qualitatively the collision effects', and his considerations 'lead to a rather general statement of the approximate distribution function which may be interpreted as of the relaxation type, demanding the free molecule distribution to tend to the continuum one exponentially with the number of collisions. Such a distribution function thus can be logically and straightforwardly constructed for a given problem and geometry. The adjustable parameters in our distribution function may still be determined by "transfer equations".' The distribution function proposed by Shen is found to be closely connected with the Krook's equation and differs slightly from the form introduced by Takao (*Rarefied Gas Dynamics*, 1961, p. 161). The use of the method is illustrated on linearized plane and cylindrical Couette flows. It seems that the proposed method may work satisfactorily for a large class of problems.

Enkenhus considers the problem of parallel flow of a reacting gas near a wall. The gas is assumed to obey the equation of state of an 'ideal dissociating gas' (Lighthill, *J. Fluid Mech.* **2**, 1957). At the wall the author applies the slip boundary conditions and on the outer edge the condition of thermodynamic equilibrium. The analysis is made for the three limiting cases: equilibrium flow, frozen flow with fully catalytic and non-catalytic wall. For all these cases the profiles of velocity, temperature and atom concentration are obtained (for O_2). The numerical solutions show 'that the surface friction may be increased by slip at large free-stream temperatures or velocities, due to the fact that the temperature jump and change in atom concentration at the wall increase the viscosity by a larger amount than the presence of slip reduces the wall velocity gradient'.

Mark examines the steady hypersonic flow of rarefied gas past a sphere using the modified boundary-layer approximation. The interaction between shock-generated vorticity and surface-generated vorticity is due to the convection of vorticity generated within the thickness of the shock wave along streamlines that enter the boundary layer and intersect the surface-generated vorticity. The velocity and enthalpy distributions in the boundary layer are determined. The shock-wave detachment distance obtained from the presented theory is in good agreement with the numerical results of Probstein & Kemp (*J. Aero/Space Sci.* **27**, 1960). It should be mentioned that this paper is a good example of the application of classical concepts of fluid dynamics to problems of rarefied gas dynamics.

Oguchi also assumes the validity of continuum flow theory and considers the problem of a sharp leading edge in rarefied hypersonic flow. The difference between this paper and the author's previous paper (in *Rarefied Gas Dynamics*, 1961) lies in the asumption of the first-order slip condition on the surface instead of the no-slip condition. According to the author the slip effect depends strongly upon the wall-to-stagnation temperature ratio.

Section 5, on experimental investigations in transition flow, is opened by a large up-to-date survey by Sherman. In the first part of his review the following problems are considered: heat transfer to a cylinder normal to supersonic flow; drag on an insulated cylinder; subsonic heat transfer from spheres; sphere drag; flow through an orifice and through a long tube, and time constants of tubes. Sherman uses a standardized method of data plotting. The Knudsen number K plays the role of an independent variable, and a dimensionless quantity F the role of a dependent variable ($F = 0$ for continuum limit and $F = 1$ for $K = \infty$). In a different type of experiment F may be the discharge coefficient, Stanton number, drag coefficient, etc. In almost all cases a deviation of about 5 % from the free molecular values of the dependent variable occurs for K ranging from about 5 to 15. Taking into account the departure of subsonic flow from continuum flow conditions, the author proposes a new and very successful correlation technique, namely, the plotting of F/F_{FM} versus F_C/F_{FM} (where the subscript C denotes the continuum limit as $M \to 0$, and the subscript FM the free-molecule limit as $Re \to 0$). The transition curve on such a plot approaches a 45° straight line (in the half-logarithmic scale) passing through the point 1·1. In the last part of the paper the author examines the limitation of wind tunnel facilities and suggests the use of free-jet tests for achieving higher Mach numbers and lower Reynolds numbers.

A second paper written by Schaaf and Maslach compares the theory with a large group of experiments. Here supersonic flow is mainly considered. The authors show that there is a large discrepancy between the experimental data and the existing theoretical determinations of the drag and heat transfer coefficients of a cross-stream cylinder in supersonic flow and the drag of a sphere in hypersonic flow.

Aroesty describes sphere-drag experiments in low density flow with Mach numbers 2, 4 and 6 and Reynolds numbers between 10 and 10,000. The experiments were performed in a wind tunnel and they revealed that the drag coefficient exhibits a small increase with decreasing Mach number for the same value of Re. In another series of experiments using the moving-model technique the author measured the force on both cooled and uncooled spheres, for a wall-to-free-stream temperature ratio T_w/T_0 ranging from 1 to 0·26.

Ashkenas measured the drag coefficient for rather high values of T_w/T_0, reaching 15 in the flow with Mach numbers from 1·8 to 4·4 and Reynolds numbers between 3 and 125. He observes that with increasing T_w/T_0 the drag increased.

Brooks and Reis describe two different techniques for measuring the drag coefficient of a circular cylinder in a subsonic flow. The first is based on radar measurements on a cloud of falling copper wires of small diameter, which were released by a sounding rocket in the upper atmosphere. The second method is based on the measurement of the damping rates of wires vibrating in a low-density chamber. The results of these experiments agree much better than would be expected with the theory proposed by the authors. The main assumption of their theoretical approach is that the same physical laws act on the body regardless of the flow regime.

A careful experimental study of hypersonic rarefied flow near the leading edge of a thin plate was performed by Chuan and Weiter in a low-density hypersonic wind tunnel of the free jet type ($M_\infty = 6$, $\lambda_\infty = 0{\cdot}01$ and $M_\infty = 8$, $\lambda_\infty = 0{\cdot}4$). The results demonstrate that existing theories cannot satisfactorily account for the observed pressure distribution along the plate.

Vidal and Wittliff report the results of experiments in the CAL 6 ft. hypersonic shock tunnel. They investigated both blunt bodies and a sharp flat plate ($M = 14$–16). A detailed comparison with different theories is made. For the blunt bodies it is shown that the data agree with the continuum theory for $K < 1$.

Valensi and Rebout give an account of preliminary measurements of stagnation heat transfer.

Brun, Facy and Trotel report the preliminary results of measurements of the drag coefficient using a lattice of nylon threads.

The problem of shock structure in front of the leading edge of a flat plate is considered by Probstein and Pan, using boundary-layer-type equations. To assure the meaningful transition to free molecule flow in the intermediate vicinity of the leading edge, the authors introduce in the Hugoniot conditions the inviscid curvature effect and also shear and heat transfer effects behind the shock. They employ the concepts of slip and temperature jump together with reflexion coefficient on the shock. All these notations are analogous to those relating to solid surfaces in rarefied gas flow. The outer boundary conditions for the viscous layer are obtained, and together with the boundary conditions at the wall allow the viscous layer equations to be solved.

Van Dyke critically examines the existing second-order hypersonic boundary-layer theories, and starting from the Lighthill constant density approximation proposes his own solution. Comparison of this solution with two sets of experimental data shows mediocre agreement with one and serious discrepancy with the other.

Only a fraction of the total number of papers have been reviewed here. I have tried faithfully to reproduce the author's formulation. A fairly large group of articles have been omitted, in which various derivations of known equations or their generalizations are discussed with no reference to experimental data. I have also omitted the papers on ionized gas flow (section 3 and part of section 1) and those in which new experimental techniques are examined (section 6).

R. Herczynski

Principles of Fluid Mechanics. By W.-H. Li and S.-H. Lam. Addison-Wesley, 1964. 374 pp. £3. 14*s*.

Principles of Fluid Mechanics. By Salamon Eskanazi. Boston: Allyn & Bacon, 1963. 478 pp. £3. 10*s*.

Fluid Mechanics for Engineers. By P. S. Barna. 2nd Edition. Butterworth, 1964. 381 pp. £2. 17*s*. 6*d*.

All three of the books under review claim to meet the needs of undergraduate students in engineering schools and might therefore be expected to have much in common; the first two, both by Americans, are indeed very similar in scope and treatment, but they differ considerably from the third book. One might infer from these books alone that a typical American undergraduate course in fluid mechanics is now strongly theoretical with relatively little practical content and with a high level of mathematical ability taken for granted, whilst in contrast a British (or Australian) course is strongly practical, is built largely on well-tried empiricisms, and the mathematical content is kept to a minimum. I know that the latter inference would be somewhat misleading, and probably the former inference is also misleading. It is indeed probable that on both sides of the Atlantic the teaching of the subject of fluid mechanics now follows more nearly similar lines than these very different books would lead one to expect.

Professors Li and Lam cover in their book the basic topics of hydrostatics, classical hydrodynamics, flow with gravity and gravity waves, viscous fluids including the derivation of the Navier–Stokes equations, the elements of laminar boundary-layer theory, a brief discussion of turbulent flows and turbulent boundary layers, compressible gas flows in one and two dimensions and associated similarity laws. Within the scope of these topics the discussion is clear and logical, fundamentals are carefully explained and a great effort is made to meet and remove difficulties that students are known to have with certain concepts. The important subject of dimensional analysis comes where it should, very early in the book. But—and inevitably there are buts—there is no wing theory and relatively little discussion of such important practical topics as propellers, pumps and turbines. One would search in vain for a discussion of the origin of circulation and therefore of lift about a body in a real fluid. The discussion of turbulent boundary layers is confined to the flow over a flat plate at zero incidence, and although adequate it shows little awareness of post-war developments in the subject. A curious lapse is the assertion on p. 117 that the drag of a streamline body is due to skin friction only and the accompanying implication that form drag is always associated with flow separation. However, the presentation and illustrations are uniformly excellent, and well chosen problems to which answers are provided at the back are liberally sprinkled throughout the text and at the ends of the chapters.

As already remarked, Professor Eskanazi's book is almost identical to that of Professors Li and Lam in scope and also shows the same omissions. The same care is shown in the development of fundamental concepts and in the effort to meet common difficulties. I was glad to see a discussion in this book of the difference between the Bernoulli equation and the energy equation for compressible flow. However, the discussion of compressible gas flow does not appear to be otherwise up to the standard of the rest of the book and is not as satisfactory or as detailed as the corresponding section of Li and Lam's book. There is no clear distinction made between Mach waves and shock waves, and oblique shocks as well as expansion fans are not discussed. The presentation and illustrations are again very good and a considerable number of examples are provided at the end of each chapter as well as useful illustrative examples in the text. Professor Eskanazi's style is somewhat ponderous but he is not obscure; one could not but warm to the acknowledgement in his preface to his wife's generosity in letting him off household chores whilst he was writing the book!

Mr Barna's book is a second edition of a book first published in 1957. The main changes made do not appear to be extensive and consist of an increase in the number of problems and an expansion of the section dealing with momentum. The list of contents suggests a most comprehensive coverage of topics including those of the two American books referred to above plus fluid metering and manometry, elements of wing theory, centrifugal and axial flow pumps and compressors as well as hydraulic turbines. However, the treatment is at a more elementary level and is largely descriptive. The book lacks the care in exposition of fundamental concepts and in the provision of clear-cut definitions that is characteristic of the other two books. Some topics are dangerously over-simplified and in spite of the elementary approach the author takes much that is far from elementary for granted. Thus, from p. 2 the reader might infer that all gases always obey the perfect gas law; from the discussion on p. 150 et seq. he might conclude that there is no interaction between the frictional and wave drag of a ship; on p. 185 he is told that flow separation always occurs a little ahead of the trailing edge (or rearmost point) of a streamline shape and he would infer that in the absence of this separation there would be no form drag; he would find the discussion of the eddy stress in the turbulent boundary layer in § 8.10 difficult to follow because of a confusion between the mean of the product of the relevant two turbulent velocity components and the product of their mean values; and on p. 254 he is suddenly introduced to Fanno curves in the temperature entropy plane without any previous definition or discussion of entropy. Nevertheless, the book is by no means without its merits. Its simplicity of approach and concern with practical matters will make an immediate appeal to the young engineering student in the first year or two of his course, and particularly good are the sections dealing with experimental topics and fluid flow machinery. The numerous examples are on the whole well chosen and a reader could get much of value from them.

For the undergraduate new to the subject, either of the two American books plus Mr Barna's book could provide a useful combination, but he would require to study them under the guidance of a good teacher. He might prefer, of course, to see whether on the well-stocked shelves of books on fluid mechanics in book shops there are single books that would more effectively and cheaply meet his needs.

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